A Brief History of the Potential Outcomes Approach to Causal inference

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1 Introduction

2 Neyman

3 Fisher

4 Rubin

Two most important early developments (1920s)

- Introduction of potential outcomes in randomized experiments by Neyman
- Introduction of randomization as the "reasoned basis" for inference by Fisher

The language and reasoning of potential outcomes in observational study settings by Rubin

- Only limited awareness of the concept of the assignment mechanism

Potential outcomes and the assignment mechanism before Nayman

- In discussion by the philosopher and economist Mill(1973) - "If a person eats of a particular dish, and dies in consequence, that is, would not have died if he had not eaten of it, people would be apt to say that eating of that dish was the source of his death"

$$\mathrm{Y}(\mathrm{eat\,dish}) = \mathrm{death}$$

$$\mathrm{Y}(\mathrm{not\,eat\,dish}) = \mathrm{not\,death}$$

Neymans's potential outcome notation in randomized experiments

- U_{ik} : potential yield,
  - $i = 1, ..., v$: variety, $k = 1, ..., m$: plot
- $a_i = \frac{1}{N}\sum_{k=1}^{m} U_{ik}$: best estimate of yield of ith variety
- x_i : sample average of the n plots actually exposed to the i^{th} variety

$$E(x_i - x_j) = a_i - a_j$$

  - $\rightarrow$ The difference in observed means, $x_i - x_j$, is unbiased for the causal estimand, $a_i - a_j$

Neymans's formalism made three contributions:

1. Explicit notation for potential outcomes
2. Implicit consideration of something like the stability assumption
3. Implicit consideration of a model for the assignment of treatments to units that corresponds to the completely randomized experiment.

Earlier hints for physical randomizing before Fisher

- "Student"(Gossett, 1923)
  - "If now the plots had been randomly placed..."
- Fisher and MacKenzie (1923)
  - "Furthermore, if all the plots were undifferentiated, as if the numbers had been mixed up and written down in random order"

Fisher's proposal to randomize treatments to units

- Neyman
  - Although he developed his notation to treat data as if they arose from what was later called a completely randomly assigned experiment,
  - He did not take the further step of proposing the necessity of physical randomization for credibly assessing causal effects.
- Fisher
  - He proposed the physical randomization of units and furthermore developed a distinct method of inference based for this special class of assignment mechanisms, that is, randomized experiments.

The observed outcome notation in observational studies for causal effects

- Fisher's p-values and Neyman's notation for potential outcomes were not used for causal inference in observational studies.
- Researchers continued building models for observed outcomes rather than thinking in terms of potential outcomes.

$$\mathrm{Y}_i^{\mathrm{obs}} = Y_i\,(W_i) = W_i \cdot Y_i(1) + (1 - W_i) \cdot Y_i(0) = \begin{cases} Y_i(0) & \text{if } W_i = 0 \\ Y_i(1) & \text{if } W_i = 1 \end{cases}$$

Early uses of potential outcomes in observational studies in social sciences

- The use of potential outcomes in the analysis of demand and supply functions specifically.
  - imaginable price (π), total demand $n(p)$, total supply $a(p)$
  - What are the functions $n(\pi)$ and $a(\pi)$?
- Similarly, Haavelmo writes:
  - total income(r), total amount for consumption($\bar{u}$)
  - $\bar{u} = \alpha * r - \beta$

Potential outcomes and the assignment mechanism in observational studies: Rubin(1974)

1. He puts the potential outcomes center stage in the analysis of causal effects, irrespective whether the study is an experimental one or an observational one.
2. He discusses the assignment mechanism in terms of the potential outcomes.